```
> with(gfun) : with(ContFrac) :
> gfun:-version( )
```

3.76 **(1)**

```
> infolevel[gfuncontfrac] := 2 :
```

> Beispiel: tan(z)

```
> Dgl := {diff(y(z), z) - 1 - y(z)^2, y(0) = 0} :
> riccati_to_cfrac(Dgl, y(z), proc(n, x) series(tan(x), x, n) end)
```

Guessing a formula
conjecture formula (on 8 coefficients)

$$\frac{z}{1+\frac{a_1 z^2}{1+\frac{\dots}{1+\frac{a_n z^2}{1+\dots}}}},\ a_n = -\frac{1}{(2n-1)(2n+1)}$$

defining a sequence H(n,z), which relates to convergence.

$$H(n, z) = \left(\frac{\partial}{\partial z} P(n, z)\right) Q(n, z) - Q(n, z)^2 - P(n, z)^2 - P(n, z) \left(\frac{\partial}{\partial z} Q(n, z)\right)$$

lemma: the conjecture holds iff there exists an unbounded i(n)
s.t., H(i(n),z) tends to 0.
(i.e., their valuations tend to infinity)
proving Limit(val(H(n,z)),n = infinity) = infinity:
- computing a recurrence for H(n,z).
(which does not conclude)

$$\dots z^8 H(n) + \dots z^4 H(n+1) + (\dots z^4 + \dots z^2)\, H(n+2) + \dots H(n+3) + \dots H(n+4) = 0$$

- reducing the recurrence order for H(n,z)...
- ... done.

$$\{-z^2 H(n) + (2n+3)^2 H(n+1),\ H(0) = -z^2\}$$

QED.

$$\frac{z}{1+\frac{a_1 z^2}{1+\frac{\dots}{1+\frac{a_n z^2}{1+\dots}}}},\ a_n = -\frac{1}{(2n-1)(2n+1)} \qquad \textbf{(2)}$$

> **oder alternativ**

```
> infolevel[gfuncontfrac] := 0 : expr_to_cfrac(tan(z), y, z)
```

(3)

$$\frac{z}{1+\frac{a_1 z^2}{1+\frac{\ldots}{1+\frac{a_n z^2}{1+\ldots}}}},\ a_n = -\frac{1}{(2n-1)(2n+1)} \tag{3}$$

> Beispiel: 1+log(1+z)

> $infolevel[gfuncontfrac] := 2 : expr\_to\_cfrac(1+\log(1+z), y, z)$

Guessing a formula
conjecture formula (on 16 coefficients)

$$1+\frac{z}{1+\frac{a_1 z}{1+\frac{\ldots}{1+\frac{a_n z}{1+\ldots}}}},\ a_n = \begin{cases} \frac{1}{4}\frac{n}{n+1} & n::even \\ \frac{1}{4}\frac{n+1}{n} & n::odd \end{cases}$$

defining a sequence H(n,z), which relates to convergence.

$$H(n, z) = \left(\frac{\partial}{\partial z} P(n, z)\right) Q(n, z)\, z - P(n, z)\left(\frac{\partial}{\partial z} Q(n, z)\right) z + \left(\frac{\partial}{\partial z} P(n, z)\right) Q(n, z) - Q(n, z)^2 - P(n, z)\left(\frac{\partial}{\partial z} Q(n, z)\right)$$

lemma: the conjecture holds iff there exists an unbounded i(n)
s.t., H(i(n),z) tends to 0.
  (i.e., their valuations tend to infinity)
proving Limit(val(H(n,z)),n = infinity) = infinity:
- computing a recurrence for H(n,z).
  (which does not conclude)

$$\ldots z^4 H(n) + \ldots H(3+n) + \ldots H(4+n) + \ldots z^2 H(n+1) + (\ldots z^2 + \ldots z) H(n+2) = 0$$

- computing a P-recurrence for H(2*n,z) using the rational formulas.
  (does not conclude either)
- reducing equation for H(2*n,z)...
- ... done.

$$\{-(n+2)^2 z^2 H(2n) + 4(2n+3)^2 H(2n+2),\ H(0) = z\}$$

QED.

(4)

$$1 + \cfrac{z}{1 + \cfrac{a_1\, z}{1 + \cfrac{\dots}{1 + \cfrac{a_n\, z}{1 + \dots}}}},\ a_n = \begin{cases} \frac{1}{4}\,\frac{n}{n+1} & n::even \\ \frac{1}{4}\,\frac{n+1}{n} & n::odd \end{cases} \tag{4}$$

>